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Proton spin structure function, g_1, with the unified evolution equations including NLO DGLAP terms and double logarithms, $ln^2(1/x)$

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Abstract. Theoretical predictions show that at low values of the Bjorken parameter x the spin structure function, g_1, is influenced by large logarithmic corrections, $\ln^2(1/x)$, which may be predominant in this region. These corrections are also partially contained in the NLO part of the standard DGLAP evolution. Here we calculate the nucleon structure function, g_1, and the gluon distribution, Δg, using the unified evolution equations written for the singlet and the non-singlet parton distributions. These equations include (i) the terms which describe the NLO DGLAP evolution and (ii) the ladder and non-ladder terms which contribute to the resummation of $\ln^2(1/x)$. Subtractions of singularities from the evolution kernels are performed so as to avoid double counting the double logarithmic contributions coming from the NLO DGLAP and the ladder and non-ladder terms. The sensitivity of the results to the factorization scheme applied is tested by introducing the DGLAP terms into the evolution equations at two different factorization schemes.

1 Introduction

The discrepancy between the theoretical expectations and the experimental data for the polarized proton has been often referred to as the "puzzle of the proton spin". This problem has been attracting the attention of the high-energy community since many years. The data obtained in 1988 by the EMC collaboration [1] showed that the total participation of the quarks in the proton spin was very small. This contradicted theoretical predictions, obtained from the well-founded Ellis–Jaffe sum rule. That sum rule connected the moments of the quark distributions to the nucleon axial coupling constants. Following that rule, the quarks should participate in about three-fifth of the total nucleon spin with the parton quark model.

New experiments emerge, and they will possibly help explaining the puzzle of the nucleon spin. The data from the region of low values of the Bjorken parameter x, $x < 10^{-3}$, will be of special importance. Theoretical predictions show that at low x the structure function, $g_1(x, Q^2)$, is influenced by large logarithmic corrections, $\ln^2(1/x)$ [2, 3]. As a consequence, large contributions to the moments of the structure functions from this region are expected. Including the region of low x into the experimental analysis will improve the estimation of the parton contributions to the nucleon spin. This will lead to a better understanding of the spin components of the nucleon.

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Analysis of the nucleon structure function at low x, including the resummation of the logarithmic corrections, was performed in [2–9]. In [4, 9–12] these logarithmic corrections were introduced for the unintegrated parton distributions. The recursive equations for their resummation were then formulated. Afterwards, the DGLAP evolution kernels calculated at the LO accuracy [4, 10] were added to these equations. That was neccessary to achieve an accurate description of the structure functions in the region of moderate and large values of x. Recently, the evolution equations for the non-singlet component of g_1 have been completed with the NLO DGLAP terms [9].

In this study we consider both the singlet and the non-singlet component of the nucleon structure function, g_1. In the singlet part we consider the NLO DGLAP evolution for the singlet quark component of g_1, $\Delta\Sigma$, and for the gluon distribution, Δg. In order to calculate $\Delta\Sigma$ and Δg, we formulate the unified evolution equations for the unintegrated parton distributions. These equations include the double logarithmic corrections, $\ln^2(1/x)$, and the NLO DGLAP terms calculated in the $\overline{\mathrm{MS}}$ factorization scheme. Following [9], subtractions of singularities are performed at the evolution kernels, so as to avoid double counting between the ladder and non-ladder terms and the NLO ones in the overlapping regions of the phase-space.

Since the singlet quark distribution, $\Delta\Sigma$, is a scheme-dependent quantity [13, 14], the choice of the factorization scheme influences its evolution. The scheme dependence disappears for the function g_1 if calculated with the standard DGLAP evolution equations. Here, the unified equations contain the extra ladder and non-ladder terms ($\sim \ln^m(1/x)$), and g_1 obtained with the unified equations may be scheme dependent. Therefore we formulate the unified evolution equations also in an alternative factorization scheme (the JET/CI scheme at the NNLO accuracy [15,16]). We then compare the results obtained from these two approaches for g_1 and Δg. Finally, our conclusions are listed.

2 Parton distributions and the DGLAP evolution

The standard DGLAP equations written for the evolution of the integrated singlet parton distributions are

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\Delta q_{\mathrm{S}}(x,Q^2) = \widetilde{\alpha_{\mathrm{s}}}(Q^2)\,(\Delta\hat{P}\otimes\Delta q_{\mathrm{S}})(x,Q^2), \quad (1)$$

where

$$\Delta q_{\mathrm{S}} = \begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}$$

is a vector consisting of the singlet quark component, $\Delta\Sigma = \sum_q(\Delta q + \Delta\bar{q})$, and of the polarized gluon distribution, Δg, with

$$\widetilde{\alpha_{\mathrm{s}}}(Q^2) \equiv \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi}.$$

These equations transform to the following integral equations:

$$f_{\mathrm{S}}(x,Q^2) = \widetilde{\alpha_{\mathrm{s}}}(Q^2)(\Delta\hat{P}\otimes\Delta q_{\mathrm{S}}^{(0)})(x) + \widetilde{\alpha_{\mathrm{s}}}(Q^2)\int_{k_0^2}^{Q^2}\frac{\mathrm{d}k^2}{k^2}\,(\Delta\hat{P}\otimes f_{\mathrm{S}})(x,k^2), \quad (2)$$

if we introduce the unintegrated parton distributions. These distributions,

$$f_{\mathrm{S}} = \begin{pmatrix} f_\Sigma \\ f_g\end{pmatrix},$$

are defined by

$$\Delta q_{\mathrm{S}}(x,Q^2) = \Delta q_{\mathrm{S}}^{(0)}(x) + \int_{k_0^2}^{Q^2}\frac{\mathrm{d}k^2}{k^2}\,f_{\mathrm{S}}(x,k^2), \quad (3)$$

with $\Delta q_{\mathrm{S}}^{(0)}(x)$ describing the contributions coming from the non-perturbative region, $Q^2 < k_0^2$. The cutoff k_0^2 is usually $\sim 1\,\mathrm{GeV}^2$. The matrix $\Delta\hat{P}$ is

$$\Delta\hat{P} = \begin{pmatrix}\Delta P_{qq}^{\mathrm{S}} & 2N_f\Delta P_{qg}\\ \Delta P_{gq} & \Delta P_{gg}\end{pmatrix}; \quad (4)$$

it contains the splitting functions as calculated for the polarized deep inelastic scattering. The constant N_f denotes the number of active flavors. Here, $N_f = 3$. The symbol $\otimes$ denotes the integral convolution of the two functions f and g,

$$(f\otimes g)(x) = \int_0^1 \mathrm{d}y\int_0^1 \mathrm{d}z\,\delta(x-yz)\,f(y)\,g(z). \quad (5)$$

The equations for the NLO DGLAP evolution of the non-singlet f_{NS} and Δq_{NS} are similar to (1). They were described in detail in [9].

A complete DGLAP analysis of the polarized parton distributions at the NLO accuracy was performed for the first time in one of the $\overline{\mathrm{MS}}$ factorization schemes. The splitting functions describing the evolution of the non-singlet components and the singlet ones were first derived in [17,18]. They are listed in detail in the appendix of [19].

The NLO DGLAP evolution of the parton distributions depends on the factorization scheme applied. In general, the scheme dependence disappears in physical observables, e.g. in the structure function, $g_1(x,Q^2)$, which is a combination of parton distributions convoluted with the Wilson coefficient functions, $\Delta C_{q,g}(x)$:

$$g_1(x,Q^2) = \frac{1}{2}\sum_{q=1}^{N_f} e_q^2\Big\{(\Delta C_q\otimes(\Delta q+\Delta\bar{q}))(x,Q^2) + (2\Delta C_g\otimes\Delta g)(x,Q^2)\Big\}, \quad (6)$$

where

$$\Delta C_q = \delta(x-1) + \widetilde{\alpha_{\mathrm{s}}}(Q^2)\,\Delta C_q^{(1)},$$
$$\Delta C_g = \widetilde{\alpha_{\mathrm{s}}}(Q^2)\,\Delta C_g^{(1)}. \quad (7)$$

The index (1) denotes that the coefficient functions are calculated at the NLO accuracy; e_q is the quark charge. The Wilson coefficients were first calculated at the $\overline{\mathrm{MS}}$ scheme [17]. Here, they were taken from [19].

Sometimes it is convenient to rewrite g_1 in terms of its non-singlet and singlet parton components, Δq_{NS} and $\Delta\Sigma$,

$$g_1(x,Q^2) = \frac{\langle e^2\rangle}{2}\Big\{\Delta C_q\otimes\Delta q_{\mathrm{NS}} + \Delta C_q\otimes\Delta\Sigma + 2N_f\Delta C_g\otimes\Delta g\Big\}, \quad (8)$$

where $\Delta q_{\mathrm{NS}} = \sum_q (e_q^2/\langle e^2\rangle - 1)(\Delta q + \Delta\bar{q})$, and $\langle e^2\rangle = \sum_q e_q^2/N_f$.

3 Scheme dependence

It is known that the $\overline{\mathrm{MS}}$ factorization does not specify a unique scheme but a family of schemes [13,20]. This is due to the ambiguities in the renormalization of operators involving γ_5 in n dimensions. In the standard $\overline{\mathrm{MS}}$

scheme [17,18] the first moment of the singlet parton distribution, $\Delta\Sigma(Q^2)$, is not conserved. The singlet moments calculated at different factorization schemes differ by a term $\widetilde{\alpha_s}(Q^2)\int_0^1 \mathrm{d}x\Delta g(x,Q^2) \sim O(\alpha_s^0)$, which can be large even at $Q^2 \to \infty$. This is a direct consequence of the axial anomaly [13,21,22]. Therefore the differences between the quark moments at different schemes can be quite large, and this makes the physical interpretation of these moments difficult.

Transforming to another factorization scheme removes some of these problems. Following [13,14,20], we introduce a family of $\overline{\mathrm{MS}}$-like factorization schemes:

$$\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}_a = (\delta\cdot\widehat{I} + \widetilde{\alpha_s}(Q^2)\widehat{Z(a)}) \otimes \begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}_{\overline{\mathrm{MS}}}. \quad (9)$$

This family is labelled by the parameter a. The index $\overline{\mathrm{MS}}$ refers to the NLO results obtained in [17–19]. The function δ is the Dirac δ-function, $\delta(x-1)$, $\widehat{I}$ is the unit matrix. The matrix $\widehat{Z(a)}$ has the form

$$\widehat{Z(a)} = \begin{pmatrix} 0 & Z(a)_{qg}\\ 0 & 0\end{pmatrix}, \quad (10)$$

where $Z(a)_{qg} = N_f[(2x-1)(a-1) + 2(1-x)]$. By a choice of the parameter a, we get a transformation of the integrated distributions to another factorization scheme. For instance, at $a=1$ we get the JET/CI factorization scheme [15,22–24], in which the parton densities are free of anomalies, and for $a=2$ we have the AB factorization scheme [25].

In the JET/CI scheme all hard effects are absorbed into the coefficient functions what results in the removal of the anomalies from the quark densities. In what follows we will use this scheme so as to test the sensitivity of our results to the factorization scheme applied. We will compare the results of the unified evolution obtained with the JET/CI scheme to those obtained with the standard $\overline{\mathrm{MS}}$ scheme.

As it was shown by Müller, Teryaev [15] and Cheng [16], the transformation to the JET/CI scheme from the $\overline{\mathrm{MS}}$ scheme has the following effect on the splitting functions:

$$\Delta P^{\mathrm{NS}}_{qq,\mathrm{JET}} - \Delta P^{\mathrm{NS}}_{qq,\overline{\mathrm{MS}}} = 0, \quad (11)$$

$$\Delta P_{gq,\mathrm{JET}} - \Delta P_{gq,\overline{\mathrm{MS}}} = 0, \quad (12)$$

$$\Delta P^{\mathrm{S}}_{qq,\mathrm{JET}} - \Delta P^{\mathrm{S}}_{qq,\overline{\mathrm{MS}}} = \widetilde{\alpha_s}(Q^2)N_f\, A\otimes\Delta P_{gq,\overline{\mathrm{MS}}}, \quad (13)$$

$$\Delta P_{gg,\mathrm{JET}} - \Delta P_{gg,\overline{\mathrm{MS}}} = -\widetilde{\alpha_s}(Q^2)N_f\, A\otimes\Delta P_{gq,\overline{\mathrm{MS}}}, \quad (14)$$

$$\begin{aligned} &2N_f(\Delta P_{qg,\mathrm{JET}} - \Delta P_{qg,\overline{\mathrm{MS}}})\\ &= \widetilde{\alpha_s}(Q^2)N_f A\otimes\Big\{\Delta P_{gg,\overline{\mathrm{MS}}} - \Delta P_{qq,\overline{\mathrm{MS}}} \quad (15)\\ &\quad - \widetilde{\alpha_s}(Q^2)N_f A\otimes\Delta P_{gq,\overline{\mathrm{MS}}}\Big\} - \widetilde{\alpha_s}(Q^2)\beta_0\, N_f\, A/2,\end{aligned}$$

where $A(x) = 2(1-x)$, $\beta_0 = 9$ at $N_f = 3$, and the kernels

$$\Delta P_{ij} \equiv \left(\Delta P^{(0)}_{ij} + \widetilde{\alpha_s}(Q^2)\Delta P^{(1)}_{ij}\right) \quad (16)$$

are defined with the indices (0) and (1) denoting the splitting functions calculated at the leading and the next-to leading accuracy, respectively.

Mueller and Teryaev obtained the analytic form of the JET/CI kernels at the $O(\alpha_s)$ accuracy (NLO) [15]. They neglected the contributions from the terms of higher orders in α_s. However, this does not give a sufficient accuracy at low x. At low x the logarithms, $\ln(1/x)$, are large, and the proper perturbative expansion should be made in powers of $\alpha_s^m \ln^n(1/x)$, and not in the powers of α_s only. To prove this, we give the following example. Analyzing the evolution kernel, $2N_f\,\Delta P_{qg,\mathrm{JET}}$, in (15), we found that its NLO part contains terms of the order $\alpha_s\ln(1/x)$, whereas its NNLO part contains the term $\alpha_s^2\ln^2(1/x)$. At low x these two terms give comparable contributions to the kernel. Therefore neglecting the NNLO contribution which couples to the large gluon distribution in the evolution equations may result in large errors during the evolution.

In order to avoid such ambiguities, we decided to calculate analytically the full form of the JET/CI kernels valid at all orders of α_s. To simplify the problem, in (13)–(15) we restricted to the $\overline{\mathrm{MS}}$ kernels calculated at the LO accuracy,

$$\Delta P_{ij,\overline{\mathrm{MS}}} \equiv \Delta P^{(0)}_{ij,\overline{\mathrm{MS}}}. \quad (17)$$

We then made the appropriate convolutions of $\Delta P_{ij,\overline{\mathrm{MS}}}$ with the transformation kernel, A, and finally got the JET/CI kernels at the NNLO accuracy,

$$\Delta P^{\mathrm{NS}}_{qq,\mathrm{JET}} - \Delta P^{\mathrm{NS}}_{qq,\overline{\mathrm{MS}}} = 0, \quad (18)$$

$$\Delta P_{gq,\mathrm{JET}} - \Delta P_{gq,\overline{\mathrm{MS}}} = 0, \quad (19)$$

$$\begin{aligned}&\Delta P^{\mathrm{S}}_{qq,\mathrm{JET}} - \Delta P^{\mathrm{S}}_{qq,\overline{\mathrm{MS}}} = -\widetilde{\alpha_s}(Q^2)\,2N_fC_F\\ &\times[3(1-x)+(2+x)\ln x], \quad (20)\end{aligned}$$

$$\begin{aligned}&\Delta P_{gg,\mathrm{JET}} - \Delta P_{gg,\overline{\mathrm{MS}}} = \widetilde{\alpha_s}(Q^2)\,2N_fC_F\\ &\times[3(1-x)+(2+x)\ln x], \quad (21)\end{aligned}$$

$$\begin{aligned}&2N_f(\Delta P_{qg,\mathrm{JET}} - \Delta P_{qg,\overline{\mathrm{MS}}}) = \widetilde{\alpha_s}(Q^2)N_f\\ &\times\{C_F[(1-x)(1-4\ln(1-x)+2\ln x)]\\ &\times C_A[(1-x)(-16+4\ln(1-x)) - 4(2+x)\ln x]\}\\ &+\left(\widetilde{\alpha_s}(Q^2)\right)^2 4N_f^2C_F \quad (22)\\ &\times\left[9(x-1) - (4x+5)\ln x - \left(1-\frac{x}{2}\right)\ln^2 x\right].\end{aligned}$$

This NNLO JET/CI evolution corresponds to the LO evolution at the $\overline{\mathrm{MS}}$ scheme.

The structure function, g_1, (cf. (8)) rewritten in terms of the parton distributions at the JET/CI scheme was

$$\begin{aligned}g_1(x,Q^2) = \frac{\langle e^2\rangle}{2}&\{\Delta q_{\mathrm{NS,JET}}(x,Q^2)\\ &+(\Delta\Sigma_{\mathrm{JET}} - \widetilde{\alpha_s}(Q^2)\,N_fA\otimes\Delta g_{\mathrm{JET}})(x,Q^2)\}. \quad (23)\end{aligned}$$

4 Unified evolution equations including resummation of double logarithms, $\ln^2(1/x)$

A complete resummation of the double logarithmic corrections, $\ln^2(1/x)$, contributing to g_1 was performed in [4].

Those corrections originated from the ladder diagrams and the non-ladder ("bremsstrahlung") diagrams [2,3,26,27], and were summed up with the recursive evolution equations written for the unintegrated parton distributions, f_i ($i = \mathrm{S}, \mathrm{NS}, g$). The integrated parton distributions were obtained from the unintegrated ones, using the relation similar to (3):

$$\Delta q_i(x, Q^2) = \Delta q_i^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{\mathrm{d}k^2}{k^2} f_i\left(x' = x\left(1 + \frac{k^2}{Q^2}\right), k^2\right), \tag{24}$$

where the phase-space was extended from Q^2 to $W^2 = Q^2(1/x - 1)$ corresponding to the total energy squared measured in the center-of-mass frame.

In [9] the unified evolution equations for f_{NS} which included both the LO DGLAP evolution terms and the double logarithms, $\ln^2(1/x)$, were completed with the NLO DGLAP terms. In order to avoid double counting the $\ln^2(1/x)$ contributions coming from the NLO terms and the ladder and non-ladder terms in the overlapping regions of the phase-space, subtractions of singularities were performed within the evolution kernels. We divided the phase-space of those equations into two regions:

(i) $k_0^2 < k^2 < Q^2$ and

(ii) $Q^2 < k^2 < Q^2/z$.

In the region (i) we kept all terms generated by the (non-ladder) double logarithmic corrections and added only the regular part of the DGLAP terms, ΔP_{reg}, i.e. that part which was not singular at $z \to 0$, hence did not generate any $\ln^2(1/x)$ contributions. In region (ii) neither LO nor NLO DGLAP terms appeared, and we had there only contributions from the ladder and the non-ladder corrections. That procedure was unique, and it used a proven result of [26,27]: the $\ln^2(1/x)$ resummation is complete after including the ladder and the non-ladder contributions.

Following [9], we write the vector equations for the unintegrated singlet distribution, f_{S},

$$\begin{aligned}
f_{\mathrm{S}}(x, Q^2) &= \widetilde{\alpha_{\mathrm{s}}}(Q^2)(\Delta\hat{P} \otimes \Delta q_{\mathrm{S}}^{(0)})(x) \\
&+ \widetilde{\alpha_{\mathrm{s}}}(Q^2) \int_{k_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} (\Delta\hat{P}_{\mathrm{reg}} \otimes f_{\mathrm{S}})(x, k^2) \\
&\qquad \text{(DGLAP)} \\
&+ \widetilde{\alpha_{\mathrm{s}}}(Q^2) \frac{4}{3} \int_x^1 \frac{\mathrm{d}z}{z} \int_{Q^2}^{Q^2/z} \frac{\mathrm{d}k^2}{k^2} f_{\mathrm{S}}\left(\frac{x}{z}, k^2\right) \\
&\qquad \text{(Ladder)} \\
&- \widetilde{\alpha_{\mathrm{s}}}(Q^2) \int_x^1 \frac{\mathrm{d}z}{z} \left(\left[\frac{\tilde{\mathbf{F}}_8}{\omega^2}\right](z) \frac{\mathbf{G}_0}{2\pi^2}\right) \int_{k_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} f_{\mathrm{S}}\left(\frac{x}{z}, k^2\right) \\
&- \widetilde{\alpha_{\mathrm{s}}}(Q^2) \int_x^1 \frac{\mathrm{d}z}{z} \int_{Q^2}^{Q^2/z} \frac{\mathrm{d}k^2}{k^2} \\
&\quad \times \left(\left[\frac{\tilde{\mathbf{F}}_8}{\omega^2}\right]\left(\frac{k^2}{Q^2} z\right) \frac{\mathbf{G}_0}{2\pi^2}\right) f_{\mathrm{S}}\left(\frac{x}{z}, k^2\right). \\
&\qquad \text{(Non-ladder)}
\end{aligned} \tag{25}$$

For a detailed form of the kernels, see Appendix A. The matrices $\mathbf{F}_8$ and $\mathbf{G}_0$ represent octet partial waves and color factors respectively. They are described in detail in Appendix A. The symbol $[\widetilde{\mathbf{F}_8/\omega^2}](z)$ denotes the inverse Mellin transform of $\mathbf{F}_8/\omega^2$:

$$\left[\widetilde{\mathbf{F}_8/\omega^2}\right](z) = \int_{\delta - \mathrm{i}\infty}^{\delta + \mathrm{i}\infty} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} z^{-\omega} \mathbf{F}_8(\omega)/\omega^2, \tag{26}$$

with the integration contour located to the right of the singularities of the function $\mathbf{F}_8(\omega)/\omega^2$.

As mentioned above, the procedure of avoiding the double counting the logarithmic contributions $\ln^2(1/x)$ is well defined and unique at the unintegrated parton distributions. It uses a proven result of [26,27]: the $\ln^2(1/x)$ resummation is complete after including the ladder and the non-ladder contributions.

However, logarithmic corrections are also present at the Wilson coefficients. They are also generated while integrating the unintegrated parton distributions, $f_{\mathrm{S,NS},g}$, over the extended phase-space, $k^2 < W^2$. In [9] we used only the regular part of the Wilson quark and gluon coefficients to avoid the double counting of those singular contributions.

However, a detailed analysis shows that there is no double counting here. Therefore cutting the singular part of the Wilson coefficients is, in general, not correct. For the gluon part of g_1 the double logarithmic corrections generated by integrating the f_g over $k_0^2 < k^2 < W^2$ contribute to g_1 with sign ($\sim -\ln x$) opposite to those present at the Wilson gluon coefficient ($\sim \ln x$). Cutting the singular part of the Wilson coefficient may then lead to a wrong behavior of the g_1 obtained at low values of x. To check this, we made also a dedicated numerical calculation (not shown).

This problem does not show up while calculating the quark part of g_1. The singular term at the Wilson quark coefficient, ΔC_q, is $\sim -\ln x$. It just enhances the singular contribution coming from the integral over the extended phase-space.

To sum up, in the present analysis we will use the full form of the quark and the gluon Wilson coefficients, $\Delta C_q^{(1)}$, $\Delta C_g^{(1)}$ both for the standard DGLAP evolution and for the unified evolution including the ladder and the non-ladder terms. The full forms of these coefficients are

$$\Delta C_q^{(1)} = \frac{4}{3}\Bigg\{(1+z^2)\left(\frac{\ln(1-z)}{(1-z)}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} + 2 + z - \frac{1+z^2}{1-z}\ln(z) - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-z)\Bigg\}, \tag{27}$$

$$\Delta C_{\mathrm{g}}^{(1)} = \frac{1}{2}\Big\{(2x-1)(\ln(1-x) - 1) - 2x\ln x + \ln x + 2(1-x)\Big\}; \tag{28}$$

they were taken from [19]. The symbol $()_+$ is defined by the following convolution. A function $f(z)$ convoluted with a function $(g(z))_+$ gives

$$\int_0^1 \mathrm{d}z\, f(z)\, (g(z))_+ = \int_0^1 \mathrm{d}z\, (f(z) - f(1))g(z). \tag{29}$$

5 Results

We numerically solved the evolution equation (25) for the singlet parton distribution, f_{S}. The NLO DGLAP terms calculated at the $\overline{\mathrm{MS}}$ scheme were taken from [19].

A simple "flat" input was used as a parameterization of the non-perturbative parton distributions at $Q_0^2 = 1\,\mathrm{GeV}^2$:

$$\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i}, \tag{30}$$

with $\eta_{u_v} = \eta_{d_v} = 3$, $\eta_{\bar{u}} = \eta_{\bar{s}} = 7$ and $\eta_g = 5$. The normalization constants N_i were determined by imposing the LO Bjorken sum rule on the $\Delta u_v^{(0)} - \Delta d_v^{(0)}$, and requiring that the first moments of all other distributions are the same as those determined from the QCD analysis [28]. It was checked that parametrization (30) combined with the unified equations gave a reasonable description of the SMC data on $g_1^{\mathrm{NS}}(x,Q^2)$ [10] and on $g_1^p(x,Q^2)$ [29]. That fit was also used in [4,30] to study the contribution of the double logarithmic corrections to the spin structure function of the proton and to its first moment.

The integrated distribution was then obtained by a numerical integration of f_{S}:

$$\Delta q_{\mathrm{S}}(x,Q^2) = \Delta q_{\mathrm{S}}^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{\mathrm{d}k^2}{k^2} f_{\mathrm{S}}\left(x' = x\left(1+\frac{k^2}{Q^2}\right), k^2\right), \tag{31}$$

following (24).

Afterwards, we made a numerical convolution of Δq_{S} with the Wilson coefficients ΔC_q, ΔC_g, in (8) in order to obtain the singlet component of the structure function, g_1, g_1^{S}. Results for the non-singlet part of g_1, g_1^{NS}, were obtained, following [9]. They were added to g_1^{S}. That sum yielded g_1. Figure 1 shows the results.

The predictions for g_1 obtained from the unified evolution including NLO DGLAP terms (DL + NLO curve) are smaller than the predictions for g_1 obtained from the pure NLO DGLAP evolution (NLO curve). This relation corresponds to the relation between the DL + LO and LO curves. This confirms that at low x the unified evolution is more singular than the standard DGLAP evolution, i.e. the DGLAP evolution may be incomplete at low x.

We made a similar observation for the gluon distribution, Δg. This distribution obtained from the DL + NLO evolution was much larger than the distribution with the standard NLO evolution. Moreover, the gluon distributions described by DL + NLO and DL + LO curves were comparable. This shows that the unified evolution of Δg is driven through the ladder and non-ladder terms which are more singular at low x than the DGLAP ones.

In order to check the sensitivity of our results to the factorization scheme applied, we then introduced the DGLAP terms at the JET/CI scheme into the evolution equations. The JET/CI kernels at the NNLO accuracy were obtained from the standard $\overline{\mathrm{MS}}$ DGLAP kernels at the LO accuracy. We transformed the input (30) into the input appropriate for the JET/CI scheme, according to the transformation rule between those schemes (9), $\Delta\Sigma_J = \Delta\Sigma_{\overline{\mathrm{MS}}} + \widetilde{\alpha_{\mathrm{s}}}(Q^2) N_f A \otimes \Delta g$, where $\Delta g = \Delta g_J = \Delta g_{\overline{\mathrm{MS}}}$.

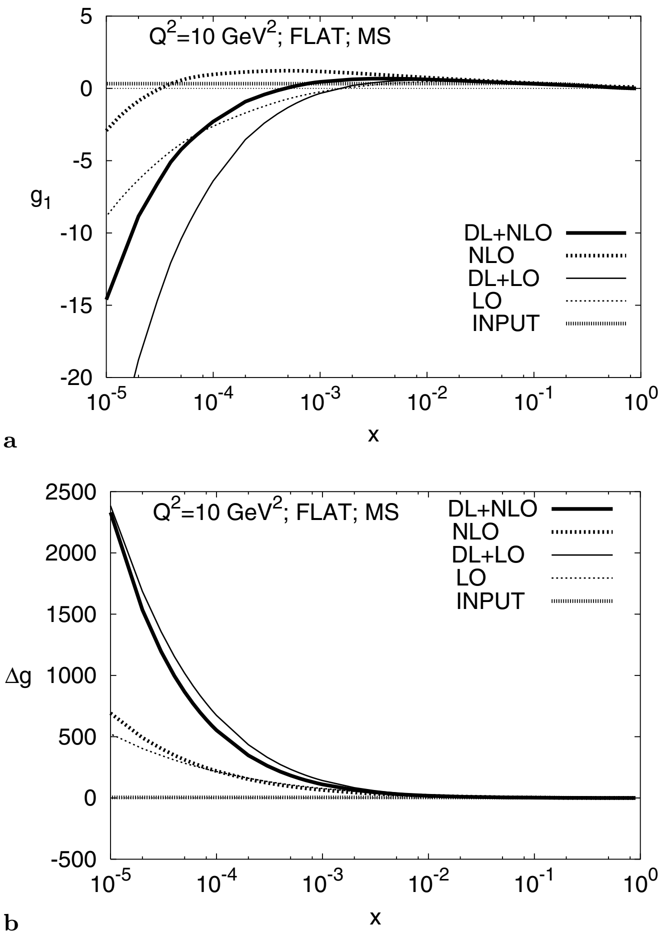

Fig. 1a,b. Proton structure function g_1^p **a** and gluon distribution Δg **b** plotted as a function of x at the fixed value $Q^2 = 10\,\mathrm{GeV}^2$. The NLO DGLAP evolution was included in the unified evolution at the $\overline{\mathrm{MS}}$ factorization scheme. The thick solid line (DL + NLO) shows the results obtained by (i) the numerical solving (25) for f_{S} with the input given (thick dotted line), (ii) the numerical integration of f_{S} performed in order to obtain Δq_{S}, Δg and for g_1^p (iii) the numerical convolution of Δq_{S}, Δg with the Wilson coefficient functions ((8), (27) and (28)). The thick dashed line (**NLO**) shows the NLO DGLAP evolution of the input. Thin lines correspond to the results: from the $\ln^2(1/x)$ resummation including DGLAP terms at the LO accuracy (solid line, DL + LO), and from the pure LO DGLAP evolution (dashed line; **LO**)

After numerical solving (25) and deriving the Δq_{S}, Δq_{NS}, we got the independent predictions for g_1 at the JET/CI scheme, which corresponded to the LO DGLAP results at the $\overline{\mathrm{MS}}$ scheme. Figure 2 shows the results.

Including the DL corrections (the ladder and non-ladder terms) into the evolution equations at the JET scheme resulted in a significant enhancement of the magnitude of g_1 predicted (DL + NNLO (JET) curve) as compared to the JET DGLAP results (NNLO (JET)) and even to the DL + LO results at the $\overline{\mathrm{MS}}$ scheme. The DL kernels coupled to the singlet gave large and negative contributions to f_{S} at low x which added to the negative con-

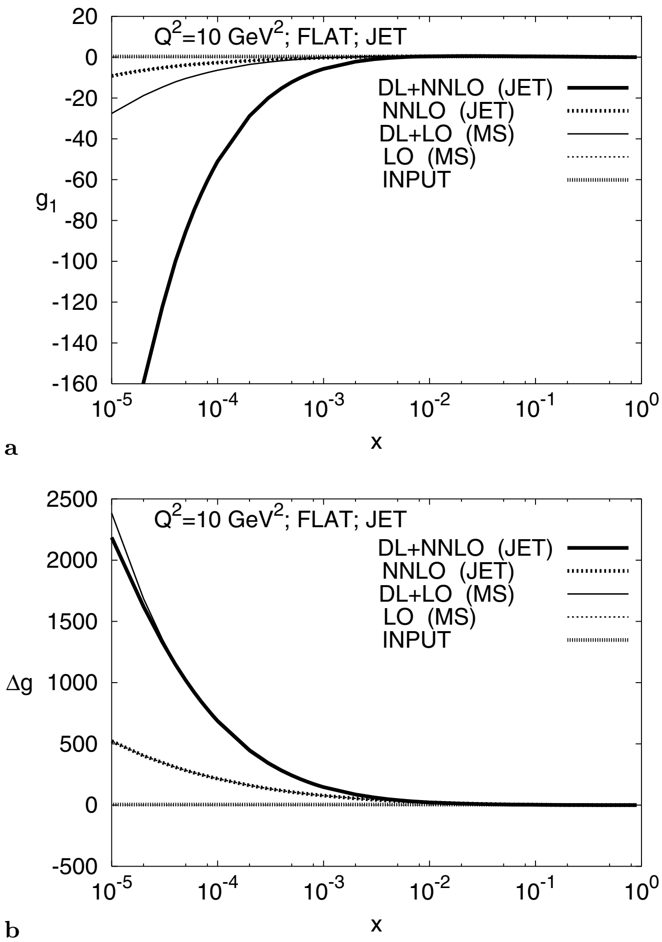

Fig. 2a,b. Proton structure function g_1^p **a** and gluon distribution Δg **b** plotted as a function of x at the fixed $Q^2 = 10\,\mathrm{GeV}^2$. The NNLO DGLAP evolution was included in the unified evolution at the JET factorization scheme. The thick solid line (DL + NNLO (**JET**)) shows the results obtained by (i) numerically solving (25) for f_S with the input given (thick dotted line) and (ii) the numerical integration of f_S performed in order to obtain Δq_S, Δg. The thick dashed line (NNLO (**JET**)) shows the NNLO DGLAP evolution of the input at the JET scheme. Thin lines correspond to the results from the $\ln^2(1/x)$ resummation including DGLAP terms at the LO accuracy at the $\overline{\mathrm{MS}}$ scheme (solid line, DL + LO (MS)), and from the pure LO DGLAP evolution at the $\overline{\mathrm{MS}}$ scheme (dashed line; **LO** (**MS**))

tributions from the JET/CI kernels. Since they were both negative, there were no cancellations between those two contributions. That lead to the enhancement observed.

The results for the gluon distributions are similar to the $\overline{\mathrm{MS}}$ scheme. The gluon distribution, Δg, obtained with the unified evolution including the JET/CI kernels is dominated by the contribution of the ladder and non-ladder terms which are more singular at low x than the DGLAP ones.

In order to check the accuracy of the JET/CI evolution, we made the following tests. We got independent predictions for g_1 and Δg from the NNLO DGLAP evolution at the JET/CI scheme and from the LO DGLAP evolution at the $\overline{\mathrm{MS}}$ scheme. In both cases the corresponding curves, NNLO (JET) and LO (MS), overlapped (Fig. 2).

Finally, we checked that our results on g_1 and Δg obtained at LO accuracy are in agreement with the corresponding results of [4].

6 Conclusions

We calculated the nucleon structure function, g_1, using the unified evolution equations written for the singlet and the non-singlet parton distributions contributing to g_1. These equations incorporated both the terms describing the NLO DGLAP evolution and the terms which contributed to the $\ln^2(1/x)$ resummation. Subtractions from the evolution kernels were performed so as to avoid double counting the $\ln^2(1/x)$ terms coming from the NLO DGLAP and from the ladder and non-ladder terms in the overlapping regions of the phase-space. The sensitivity of the results to the factorization scheme applied was tested by introducing the NLO DGLAP terms in two different schemes into the evolution equations.

In both schemes we got an enhancement of the magnitude of g_1 and Δg obtained with the unified evolution as compared to the pure DGLAP results. This enhancement was significant for g_1 calculated at the JET/CI factorization scheme. The behavior of the gluon distribution with the unified evolution at low x was clearly dominated by the large contribution of the ladder and non-ladder terms.

We noticed that the singularities at low x generated during the integration of the unintegrated parton distributions over the extended phase-space ($k^2 < W^2$) did not reproduce the singularities present at the Wilson coefficients. Therefore there is no risk of double counting these singular contributions while calculating g_1.

We also discussed and showed possible ambiguities, when introducing the DGLAP evolution in different factorization schemes into the unified evolution equations. They are due to the large contributions from $\ln^m x$ which require making a perturbative expansion of the kernels in powers of $\alpha_\mathrm{s}^n \ln^m x$ and not in powers of α_s only.

To sum up, our observations suggest that the standard DGLAP evolution is not complete at low x, where the effects of the $\ln^2(x)$ resummation are large and therefore cannot be neglected.

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Appendix A

Here a brief description of the evolution kernels of (25) is given. The DGLAP kernels were taken from [19]. The full DGLAP kernel ΔP includes both the LO and NLO terms:

$$\Delta P = \Delta P^{(0)} + \widetilde{\alpha_s}(Q^2)\, \Delta P^{(1)}. \qquad (32)$$

Only the regular part of the full kernel is included into the homogeneous term,

$$\widetilde{\alpha_s}(Q^2) \int_{k_0^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \, (\Delta P_{\mathrm{reg}} \otimes f_{\mathrm{NS}})(x, k^2),$$

appearing in (25). This is to avoid double counting the logarithmic contributions coming from the NLO terms and the non-ladder terms in the region of the phase-space, $k_0^2 < k^2 < Q^2$.

Ladder kernels corresponding to the LO DGLAP kernels at the longitudinal momentum transfer, $z = 0$ [4] generate the double logarithmic corrections in the region of $Q^2 < k^2 < Q^2/z$.

Non-ladder kernels were obtained in [4] from the infrared evolution equations written for the singlet partial waves $\mathbf{F}_0$, $\mathbf{F}_8$ [2,3,26,27]. In [4] we noticed that extending the kernel of the double logarithmic evolution equations from the ladder one,

$$\widetilde{\alpha_s}(Q^2) \Delta P_{qq}/\omega, \qquad (33)$$

to the modified one,

$$\widetilde{\alpha_s}(Q^2) \left(\Delta P_{qq}/\omega - (\mathbf{F}_8(\omega)\, \mathbf{G}_0)_{qq}/(2\pi^2\omega^2) \right), \qquad (34)$$

gave a proper anomalous dimension as derived from the infrared evolution equations.

The matrix $\mathbf{G}_0$ contained color factors resulting from attaching the soft gluon to the external legs of the scattering amplitude:

$$\mathbf{G}_0 = \begin{pmatrix} \frac{N^2-1}{2N} & 0 \\ 0 & N \end{pmatrix}, \qquad (35)$$

where N was the number of colors.

Further, it was checked that the Born approximation of $\mathbf{F}_8$,

$$\mathbf{F}_8^{\mathrm{Born}}(\omega) \approx 8\pi^2 \widetilde{\alpha_s}(Q^2) \frac{\mathbf{M}_8}{\omega}, \qquad (36)$$

gave accurate results for the DL evolution. The matrix $\mathbf{M}_8$ was a splitting function matrix in the color octet t-channel:

$$\mathbf{M}_8 = \begin{pmatrix} -\frac{1}{2N} & -\frac{N_f}{2} \\ N & 2N \end{pmatrix}. \qquad (37)$$

The inverse Mellin transform of $\mathbf{F}_8^{\mathrm{Born}}(\omega)$ then reads

$$\left[\frac{\tilde{\mathbf{F}}_8^{\mathrm{Born}}}{\omega^2} \right](z) = 4\pi^2 \widetilde{\alpha_s}(Q^2) \mathbf{M}_8 \ln^2(z). \qquad (38)$$

The evolution equation (25) includes the non-ladder corrections in the Born approximation (38).

References

1. J. Ashman et al., EMC, Phys. Lett. B **206**, 364 (1988)
2. J. Bartels, B.I. Ermolaev, M.G. Ryskin, Z. Phys. C **70**, 273 (1996)
3. J. Bartels, B.I. Ermolaev, M.G. Ryskin, Z. Phys. C **72**, 627 (1996)
4. J. Kwieciński, B. Ziaja, Phys. Rev. D **60**, 054004 (1999)
5. J. Blümlein, A. Vogt, Phys. Lett. B **370**, 149 (1996)
6. J. Blümlein, A. Vogt, Acta Phys. Pol. B **27**, 1309 (1996)
7. J. Blümlein, A. Vogt, Phys. Lett. B **386**, 350 (1996)
8. J. Blümlein, Lectures given at Ringberg Workshop: New Trends in HERA Physics (1999) Ringberg Castle, Tegernsee, Germany, 30 May–4 June 1999; in Tegernsee 1999, New trends in HERA physics 42-57, edited by G. Grindhammer, B. Kniehl, G. Kramer (Springer, Berlin 1999), hep-ph/9909449
9. B. Ziaja, Phys. Rev. D **66**, 114017 (2002)
10. B. Badełek, J. Kwieciński, Phys. Lett. B **418**, 229 (1998)
11. J. Kwieciński, B. Ziaja, in Proceedings of the Workshop Physics with polarized protons at HERA, DESY March–September 1997, edited by A. De Roeck, T. Gehrmann, DESY Proceedings 1998, 1997
12. S.I. Manayenkov, M.G. Ryskin, in Proceedings of the Workshop Physics with polarized protons at HERA, DESY March–September 1997, edited by A. De Roeck, T. Gehrmann, DESY Proceedings 1998, 1997
13. E. Leader, A.V. Sidorov, D.B. Stamenov, Phys. Lett. B **445**, 232 (1998)
14. E.B. Zijlstra, W.L. van Neerven, Nucl. Phys. B **417**, 61 (1994)
15. D. Mueller, O.V. Teryaev, Phys. Rev. D **56**, 2607 (1997)
16. H.-Y. Cheng, Phys. Lett. B **427**, 371 (1998)
17. R. Mertig, W.L. van Neerven, Z. Phys. C **70**, 637 (1996)
18. W. Vogelsang, Phys. Rev. D **54**, 2023 (1996)
19. B. Lampe, E. Reya, Phys. Rept. **332**, 1 (2000)
20. E. Leader, J. Phys. G **25**, 1557 (1999)
21. R.D. Carlitz, J.C. Collins, A.H. Mueller, Phys. Lett. B **214**, 229 (1988)
22. H.-Y. Cheng, Chin. J. Phys. **38**, 753 (2000)
23. R.D. Carlitz, J.C. Collins, A.H. Mueller, Phys. Lett. B **214**, 229 (1988)
24. M. Anselmino, A.V. Efremov, E. Leader, Phys. Rept. **261**, 1 (1995)
25. R.D. Ball, S. Forte, G. Ridolfi, Phys. Lett. B **378**, 255 (1996)
26. R. Kirschner, L.N. Lipatov, Nucl. Phys. B **213**, 122 (1983)
27. R. Kirschner, Z. Phys. C **67**, 459 (1995)
28. M. Stratmann, hep-ph/9710379 (1997)
29. B. Badełek, J. Kiryluk, J. Kwieciński, Phys. Rev. D **61**, 014009 (2000)
30. B. Ziaja, Acta Phys. Polon. B **32**, 2863 (2001)